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Bouncing Cosmology in Three Dimensions

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Abstract

We consider a dynamical two-brane in a four dimensional black hole background with scalar hair. At high temperature this black hole goes through a phase transition by radiating away the scalar. The end phase is a topological adS-Schwarzschild black hole. We argue here that for a sufficiently low temperature, the brane motion in this geometry is non-singular. This results in a universe which passes over from a contracting phase to an expanding one without reaching a singularity.

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In standard cosmology, the universe usually starts or/and ends in a cosmological singularity. Since near the singularity the gravitational interaction is strong, classical general relativity breaks down. Consequently, in order to study the fate of these singularities, one needs to look for a still illusive theory of quantum gravity. It is therefore of interest to search for a non-singular cosmological model by some how evading Penrose-Hawking singularity theorem [1]. If this is possible, one can at least perform some further computations around these cosmological backgrounds while trusting the perturbative theory of gravity.

In the context of brane-world models, where the brane moves in a higher dimensional bulk, we often get exotic cosmologies on the brane (see [2, 3] for example). Since, in principle, in this scenario, bulk geometry can contribute a negative energy density on the brane [4], it may be possible to construct some non-singular cosmological models. Indeed, some models of this nature were constructed in [5, 6, 7, 8]. In particular, in [5], a non-singular cosmology was found by considering a three-brane in an electrically charged adS black hole background [9]. Here, the brane contracts to a finite size before expanding again. This happens because the background charge introduces a negative energy density on the brane world volume; hence preventing it to fall into the singularity. However, the bounce occurs inside the outer horizon of the black hole. As a result, regardless of the initial energy of the brane, the bulk space time collapses due to the back reaction at the bounce [10].

In our effort to search for other examples of non-singular cosmology, in this Letter, we study another model of bounce in three dimensions. As we will show below, in certain region of the parameter space of the bulk geometry, the brane makes a smooth transition from a contracting phase to an expanding phase without ever reaching to a singularity. The model of our interest consists of black holes in four dimensional gravity with self-interacting scalar field and a negative cosmological constant. The solutions are of topology $\mathbf{R}^2 \times \Sigma$ where Σ represents a two dimensional manifold with a constant negative curvature. These holes are parametrised by a single parameter associated with the mass of the black holes. Interestingly enough, the mass of the black hole here can take negative values without producing a naked singularity. We will take an advantage of this fact to construct a bouncing brane cosmology by considering a dynamical two-brane in this geometry. In the following, we describe the model in some detail and, subsequently, analyse the resulting cosmology.

Let us start by considering the action

$$S = \int d^4x\sqrt{-g}\Big[\frac{R+6/l^2}{16\pi} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{3}{4\pi l^2}\sinh^2(\frac{\sqrt{4\pi}\phi}{\sqrt{3}})\Big]. \tag{1}$$

Around the global maximum ($\phi = 0$), the scalar satisfies the Breitenlohner-Friedman bound [11] ensuring perturbative stability of the adS space-time. A black hole solution of this action is given by [12]

$$ds^2 = \frac{r(r+2m)}{(r+m)^2}\Big[-\Big(\frac{r^2}{l^2} - (1+\frac{m}{r})^2\Big)dt^2 + \Big(\frac{r^2}{l^2} - (1+\frac{m}{r})^2\Big)^{-1}dr^2 + r^2 d\sigma^2\Big], \tag{2}$$

with the scalar

$$\phi = \sqrt{\frac{3}{4\pi}}\mathrm{Arctanh}\Big(\frac{m}{r+m}\Big). \tag{3}$$

In (2), $d\sigma^2$ represents an element of a two dimensional manifold with a negative constant curvature (Σ). The mass of the black hole is given by

$$M = \frac{\sigma}{4\pi} m, \tag{4}$$

where σ is the area on Σ. For $m > 0$, the solution has a singularity at $r = 0$. This singularity is hidden behind a horizon. For $m < -l/4$, the metric has a naked singularity. However, for $-l/4 < m < 0$, the singularity at $r = -2m$ is surrounded by three horizons with the outermost horizon is at

$$r_+ = \frac{l}{2}\Big(1 + \sqrt{1 + \frac{4m}{l}}\Big). \tag{5}$$

Note that the origin of the radial coordinate here should be taken at $r = -2m$. The temperature of the black hole is given by

$$T = \frac{1}{2\pi l}\Big(\frac{2r_+}{l} - 1\Big). \tag{6}$$

The entropy is proportional to the horizon area and is

$$S = \frac{\sigma}{4} \frac{{r_+}^3 (r_+ + 2m)}{(r_+ + m)^2}. \tag{7}$$

One of the interesting properties of this black hole is that it undergoes a phase transition. This happens as we increase the temperature above a critical temperature given by $T_c = 1/2\pi l$. At this point $m \to 0$ and $r_+ \to l$. For temperature above $T = T_c$, the black hole radiates away its scalar hair and decays to a topological Schwarzschild-adS black hole with metric

$$ds^2 = -\Big(\frac{r^2}{l^2} - 1 - \frac{2m}{r}\Big)dt^2 + \Big(\frac{r^2}{l^2} - 1 - \frac{2m}{r}\Big)^{-1} dr^2 + r^2 d\sigma^2, \tag{8}$$

with temperature

$$T = \frac{1}{4\pi}\Big(\frac{3r_+}{l} - \frac{1}{r_+}\Big), \tag{9}$$

and entropy

$$S = \frac{\sigma r_+^2}{4}. \tag{10}$$

Here r_+ is the horizon location of (5) and should not be confused with the r_+ appearing in (2). Note that both the metric (2) and (8) have same asymptotic geometry. Furthermore, as can be seen from (7) and (10), the entropy changes continuously around the transition point. It is also possible to define an order parameter which changes continuously to zero as we increase the temperature above $T = T_c$[12].

We would now like to consider a two-brane with three dimensional world volume moving in this background. We will assume that the brane is much lighter than the bulk. Consequently, any back reaction to the background geometry will be neglected. The world volume action of the brane is

$$S_b = -\int d^3x \sqrt{-\gamma}\hat{V}(\phi), \tag{11}$$

where γ_{ij} is the induced metric on the brane and $\hat{V}(\phi)$ is the potential on the brane which we will determine shortly.

Let us assume that the bulk metric has a general form

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + R^2(r)d\sigma^2. \tag{12}$$

If we write the induced brane metric as

$$ds_b^2 = \gamma_{ij}dx^i dx^j = -d\tau^2 + R^2(\tau)d\sigma^2, \tag{13}$$

then it follows immediately from (12) and (13) that

$$\Big(\frac{\partial \tau}{\partial t}\Big)^2 = \frac{A}{1 + B\Big(\frac{\partial r}{\partial \tau}\Big)^2}. \tag{14}$$

Finally, using (14) in the Israel junction condition (as discussed, for example, in [13]) we get

$$\frac{1}{2}\Big(\frac{dR}{d\tau}\Big)^2 + F(R) = 0, \tag{15}$$

where $F(R)$ is given by

$$F(R) = \frac{1}{2B}\Big(\frac{dR}{dr}\Big)^2 - \frac{\hat{V}^2 R^2}{32}. \tag{16}$$

Above equation can also be expressed as

$$\frac{1}{2}\Big(\frac{dr}{d\tau}\Big)^2 + U(r) = 0, \tag{17}$$

where the potential term

$$U(r) = \frac{1}{2B} - \frac{\hat{V}^2 R^2}{32 R'^2}. \tag{18}$$

In writing (17), we used the fact that $R' = dR/dr$ is nonzero everywhere in the region $r > -2m$ and it goes to a constant value at large r. Eqn. (17) can be thought of as a Hamiltonian constraint for a zero energy classical particle.

We now turn our attention to the brane potential $\hat{V}(\phi)$. Firstly, $\hat{V}$ can not be chosen arbitrarily. It is fixed by the boundary condition on the bulk scalar field at the brane. Following [13], the boundary condition on this scalar at the wall can be written as

$$\{n.\partial\phi\} = \frac{d\hat{V}}{d\phi}, \tag{19}$$

where n is the unit normal at the boundary pointing towards the bulk and the curly brackets denote summation over both sides of the wall. The above equation can be simplified [13] using (15) and (2) to get

$$\frac{d\phi}{dR} = -\frac{2}{R}\frac{d}{d\phi}\log \hat{V}. \tag{20}$$

For the scalar profile (3), one can solve (20) explicitly with the result

$$\hat{V} = \lambda \left[\frac{(3m^2 + 3mr + r^2)^2}{r(r+2m)^3} \right]^{\frac{1}{16\pi}}, \tag{21}$$

where, λ is an integration constant and related to the brane tension. This can be seen by taking $r \to \infty$ limit of (21) and comparing it with the boundary action S_b given in eqn. (11). Though not very illuminating, $\hat{V}$ can indeed be expressed as a function of the scalar ϕ only. It is given by

$$\hat{V}(\tilde{\phi}) = \lambda \left[\frac{\coth^2\tilde{\phi} + \coth\,\tilde{\phi} + 1}{\mathrm{cosech}\,\tilde{\phi}\;(\coth\,\tilde{\phi} + 1)} \right]^{\frac{1}{8\pi}}, \tag{22}$$

where $\tilde{\phi} = \sqrt{4\pi/3}\phi$. Note that the explicit dependence of the black hole mass term m has disappeared in (22). We therefore conclude that even if in (11), we have introduced $\hat{V}(\phi)$ in somewhat adhoc manner, the potential gets determined (upto a constant λ) via the bulk scalar boundary condition on the brane.

For our metric (2), $U(r)$ in (18) can be explicitly computed. It is given by

$$U(r) = \frac{1}{2}(r+m)^2 \left[\frac{\frac{r^2}{l^2} - (1 + \frac{m}{r})^2}{r(r+2m)} - \frac{\lambda^2 (r^2 + 3mr + 3m^2)^{\frac{1}{4\pi}}}{16(r(r+2m)^3)^{\frac{1}{8\pi}}} \frac{r^2(r+2m)^2}{(r^2 + 3mr + 3m^2)^2} \right]. \tag{23}$$

Following our earlier discussion, we note that for higher temperature, the black hole goes through a phase transition. Hence, equation (17) describes the correct behaviour of the brane universe only for $T < T_c$. For temperature $T > T_c$, the bulk is a topological Schwarzschild-adS black hole. Therefore, the effective potential $U(r)$ modifies to

$$\tilde{U}(r) = \frac{1}{2}\Big[-1 + \Big(\frac{1}{l^2} - \frac{\lambda^2}{16} \Big) r^2 - \frac{2m}{r} \Big]. \tag{24}$$

This follows again from the Israel junction condition when the bulk is given by metric (8).

We would now be interested in solving the eqn. (17) for $T < T_c$ where the black hole mass is negative and in the region $-l/4 < m < 0$. Since, R' is a smooth non-zero function in the range $r > -2m$, the cosmological nature of the brane can be inferred from (17) and (23). First of all, this behaviour crucially depends on the signature of $\lambda^2 - 16/l^2$. We will mainly be interested in the case when this quantity is positive.

As it is not possible to solve (17) analytically in a compact form, we will have to resort to solving it by numerical means. However, before we do so, we will try to understand the qualitative nature of the solution from the potential (23) itself. Since for large r,

$$U(r) \sim \Big(\frac{1}{l^2} - \frac{\lambda^2}{16} \Big) r^2, \tag{25}$$

for $\lambda^2 > 16/l^2$, we get the deSitter like solutions. On the other hand, near the singularity at $r = -2m$

$$U(r) \sim m \Big(\frac{1}{16} - \frac{m^2}{l^2} \Big) (r+2m)^{-1}. \tag{26}$$

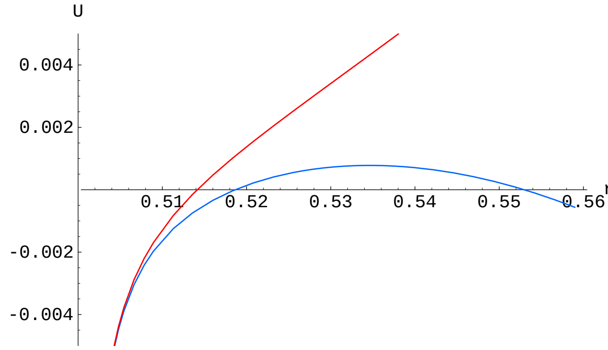

Figure 1: The bottom line is the potential $U(r)$. The largest r for which $U(r) = 0$ is the location of the bounce. The largest value of r at which the top line crosses the horizontal axis corresponds to the outer horizon. These plots are for $l = 1, \lambda = 4.1$ and $m = -.2498$.

which is independent of the tension of the brane. One of the most interesting property of the potential is that for certain range of the mass of the black hole there is a turning point. This can be seen clearly in fig.1. This point corresponds to the largest solution r_b of the equation $U(r) = 0$. The expression of $U(r)$ is given in (23). As this point is outside the outer most horizon of the black hole, the brane will bounce back even before reaching the horizon. This will result in a bouncing cosmology where the universe reaches a minimum size $R(r_b)$ and then expands again. It is straightforward to integrate (17) numerically. The result is shown in fig.2.

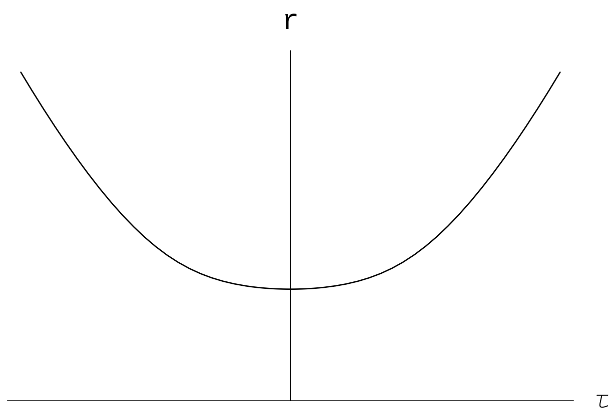

Figure 2: Solution of eqn.(17). Time is chosen such that the bounce is at $\tau = 0$.

In case of λ^2 sufficiently less than $16/l^2$, all the solutions are singular. In this case, the brane can start at a finite radius and can fall into the black hole singularity, resulting in a singular brane universe. Typical nature of the potential is shown in fig.3.

For fixed l and λ, if we increase the temperature of the black hole beyond T_c, due to the phase transition, the effective potential felt by the brane is given by eqn.(24). This always leads to a singular cosmology as the brane collapses into the adS-Schwarzschild singularity (Discussions of this can be found, for example, in [14]).

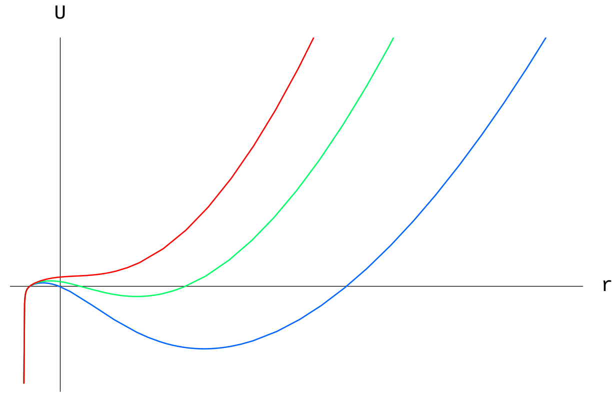

Figure 3: $U(r)$ for $\lambda^2 < 16/l^2$. In λ is sufficiently less than $16/l^2$ brane starts at a finite radius and falls into the black hole singularity. Plots are for different values of λ^2.

To conclude, we therefore have the following picture. Consider a brane with fixed energy density (larger than the scale set by the adS curvature) in the asymptotic region of a topological adS-Schwarzschild geometry. As we decrease the temperature of the black hole below a critical value, the bulk goes through a phase transition. In this phase, the geometry is that of a black hole with scalar hair. Consequently, the singular cosmology of the brane universe at high temperature modifies to a bouncing cosmology at low temperature. Note that, unlike some other models, this bounce takes place even before the brane reaches the horizon of the bulk. Before we end, we would like to point out that our conclusion is based on a completely classical analysis. This picture might get modified in several way. Firstly, instead of an empty brane, one expects the brane to carry additional matter. This will change the behaviour of the brane universe in a significant way. Secondly, various instabilitis may creep in when the model is treated semi-classically. We leave all these issues for future considerations.

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References

[1] S.W. Hawking and G.F.R. Ellis, 'The large scale structure of space time', Cambridge University Press, Cambridge, England (1973).

[2] R. Maartens, Living Rev. Rel. 7 (2004) 7, gr-qc/0312059.

[3] A. Kehagias and E. Kiritsis, JHEP 9911 (1999) 022, hep-th/9910174.

[4] D.N. Vollick, Gen. Rel. Grav. 34 (2002) 1, hep-th/0004064.

[5] S. Mukherji and M. Peloso, Phys. Lett. B547 (2002) 297, hep-th/0205180.

[6] A. Biswas, S. Mukherji and S.S. Pal, Int. J. Mod. Phys. A19 (2004) 557, hep-th/0301144.

[7] N. Okuyama and K. Maeda, Phys. Rev. D70 (2004) 064030, hep-th/0405077.

[8] C.P. Burgess, F. Quevedo, R. Rabadan, G. Tasinato, I. Zavala, JCAP 0402 (2004) 008, hep-th/0310122.

[9] C. Bercelo and M. Visser, Phys. Lett. B482, (2000) 183, hep-th/0004056.

[10] J.L. Hovdebo and R.C. Myers, JCAP 0311 (2003) 012, hep-th/0308088.

[11] P. Breitenlohner and D.Z. Freedman, Phys. Lett. B115 (1982) 197; Annals. Phys. 144 (1982) 249.

[12] C. Martinez, R. Troncoso and J. Zanelli, Phys. Rev. D70 (2004) 084035, hep-th/0406111.

[13] H.A. Chamblin and H.S. Reall, Nucl. Phys. B562 (1999) 133, hep-th/9903225.

[14] A.C. Petkou and G. Siopsis, JHEP 0202 (2002) 045, hep-th/0111085.